

# Brane inflation with dark reheating

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(Dated: February 2, 2008)

In brane world scenarios inflationary vacuum energy may escape into the higher dimensional bulk leaving behind a dark radiation effect on the brane. The paper analyses the damping of an inflaton by massless bulk scalar radiation and the production of dark radiation in a Randall-Sundrum type of model.

## I. INTRODUCTION

In brane world scenarios, matter fields are confined to a four dimensional hypersurface, or brane, embedded in a higher dimensional bulk. Brane cosmology can reproduce the most important features of the usual four dimensional cosmology, but sometimes the extra dimensions make themselves known and one such effect is considered here.

Many brane world scenarios are built around the Randall-Sundrum models [1, 2], where there is a five dimensional anti-de Sitter bulk spacetime and either a double or a single brane. Inflationary solutions to the single brane Randall-Sundrum model can be obtained by simply ‘detuning’ the vacuum energy of the brane [3, 4]. This can be done with the extra vacuum energy of a brane or a bulk inflaton field. The former case is called matter field inflation [5].

In 4-dimensional models, hot big bang radiation is produced by the decay of the inflaton field. In 5 dimensions, the inflaton can decay into ordinary matter fields or into bulk fields. In most models of inflation, the coupling between the inflaton and other matter fields is very small, and may be smaller even than the coupling to gravity or other bulk fields. Some of the inflaton’s vacuum energy can easily end up in the bulk.

In the most extreme case, nearly all of the vacuum energy of the inflaton field may decay into bulk radiation. This happens according to one scenario, put forward by Enqvist et al. [6, 7]. In their scenario, the entropy of the universe arises from fields associated with the flat directions of the Minimally Supersymmetric Standard Model (MSSM). For this scenario to work, the energy released onto the brane by these fields has to dominate over the energy produced by the decay of the inflaton, which can happen if the inflaton decays into bulk degrees of freedom.

The transfer of energy from a brane to the bulk is always associated with a geometrical effect on the brane which mimics a radiation source term in the cosmological evolution equations [8, 9, 10, 11, 12, 13, 14, 15]. The size of this dark radiation term in relation to the real radiation is an important feature, because we have strong evidence that the real radiation provides the dominant proportion of the radiation density driving the expansion rate at the nucleosynthesis era.

The purpose of this paper is to construct a simple model of the inflaton decay process with a single brane and a massless scalar bulk field. The bulk field might occur as a hypothetical superpartner of the graviton or a bulk dilaton field, for example. We shall construct the reheating terms in the inflaton’s equation of motion and solve for the case where bulk radiation is the predominant form of reheating. We shall then examine the cosmological evolution to determine the residual amount of dark radiation.

The action for the model takes the form

$$S_g = -\frac{1}{\kappa_5^2} \int_{\mathcal{M}} (R - 2\Lambda) dv + \frac{2}{\kappa_5^2} \int_{\partial\mathcal{M}} K dv + \int_{\mathcal{M}} \mathcal{L}_b dv + \int_{\partial\mathcal{M}} (\mathcal{L}_m - \sigma) dv \quad (1)$$

where we have a curved space  $\mathcal{M}$  with scalar curvature  $R$  and a boundary  $\partial\mathcal{M}$  with extrinsic curvature  $K$ . The covariant volume integral is denoted by  $dv$ . We will take coordinates  $x^\mu$ ,  $\mu = 0, 1, 2, 3$ , along the brane and  $x^5 = z$  perpendicular to the brane. In the final state after inflation, the vacuum energy  $\sigma$  and the 5-dimensional cosmological constant  $\Lambda$  satisfy the fine tuning relationship  $6\Lambda = \kappa_5^4 \sigma^2$  [1, 2]. The effective four dimensional Newton’s constant  $G$  is given by  $48\pi G = \kappa_5^4 \sigma$ .

The inflaton field  $\phi(x)$  is restricted to the brane, with Lagrangian density

$$\mathcal{L}_m = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi). \quad (2)$$

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In the bulk, there is a massless conformally coupled scalar field  $\Phi(x)$  with Lagrangian density

$$\mathcal{L}_b = -\frac{1}{2}(\partial_a \Phi)^2 - \frac{3}{32}R\Phi^2. \quad (3)$$

Conformally invariant boundary conditions fix the normal derivative of the Bulk scalar. We consider an interaction which allows  $\phi$  to spontaneously decay into  $\Phi$ . Since the  $\phi$  field is confined on the brane, the two fields can only interact at  $z = 0$ . A simple model for the Lagrangian density of the interaction is

$$\mathcal{L}_I = -\frac{\lambda}{2}\phi(x)\Phi(x)^2\delta(z), \quad (4)$$

where  $\lambda$  is the coupling constant [7].

The effect of the radiation on the inflaton field equation is a damping term. The general form of this term is identical to the four dimensional case which can be found in ref. [16]. For a flat brane,

$$\frac{d^2\phi}{dt^2} + \frac{dV}{d\phi} = J \quad (5)$$

where the leading order contribution to the dissipation is

$$J(t) = \lambda^2 \text{Im} \int d^4x' G_F(x, x')^2 \phi(t') \theta(t - t') \Big|_{z=z'=0}, \quad (6)$$

and  $G_F(x, x')$  is the Feynman propagator of the bulk field  $\Phi$  with Robin boundary conditions on the brane.

For flat extra dimensions,

$$G_F(x, x') = \int \frac{d^3k}{(2\pi)^3} \frac{dq}{2\pi} 4 \cos qz \cos qz' e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \frac{i}{2\omega} e^{-i\omega(t-t')}, \quad (7)$$

where  $\omega = (\vec{k}^2 + q^2)^{1/2}$ . The intergral (6) is divergent, but we can apply dimensional regularisation. In five dimensions we find

$$J(t) = -\frac{\lambda^2}{512\pi^2} \ln\left(\frac{1}{\mu} \frac{d}{dt}\right) \left(\frac{d}{dt}\right)^2 \phi(t), \quad (8)$$

where  $\mu$  is the one-loop renormalisation scale. The same renormalisation scale appears in the one-loop corrections to the potential. In order for these corrections to remain relatively small,  $\mu$  has to be large compared to other mass scales in potential.

The logarithmic time derivative appearing in eq. (8) is a feature of radiation back reaction effects in odd spacetime dimensions [17]. For practical applications, we can use an integral kernel to define the logarithmic derivative (see [17]),

$$\ln\left(\frac{1}{\mu} \frac{d}{dt}\right) \phi = - \int_{-\infty}^t \ln(e^\gamma \mu(t-t')) \frac{d\phi}{dt'} dt' \quad (9)$$

where  $\gamma$  is Euler's constant.

This result which we have found applies to flat branes and flat extra dimensions. However, we can make some simple generalisations. In the first place, for the special case of the conformal scalar field, we can use the same result for any bulk spacetime which is conformal to flat space. In particular, the result also applies to anti-de Sitter extra dimensions.

Another generalisation can be made if the brane is spatially homogeneous with expansion rate  $H$  in the range  $H^2 \ll \dot{\phi}$ . In four dimensions, the amplitude of density perturbations is of order  $H^2/\dot{\phi}$ , and  $H^2 \ll \dot{\phi}$  is consistent with having small amplitude density fluctuations. In this case the time evolution of the inflaton is the dominant effect producing the dissipative term in the inflaton equation and we can use the flat space result (8).

The complete form for inflaton equation of motion on an expanding brane with  $H^2 \ll \dot{\phi}$  is

$$\frac{d^2\phi}{dt^2} + 3H \frac{d\phi}{dt} + \frac{dV}{d\phi} = -c \ln\left(\frac{1}{\mu} \frac{d}{dt}\right) \frac{d^2\phi}{dt^2} \quad (10)$$

where  $c = \lambda^2/(512\pi^2)$ . The form of the damping term is strongly dependent on the fact that we have not introduced any explicit mass terms into the bulk fields. When the same calculation is repeated for a bulk field of mass  $M$ ,

there is no damping of the inflaton when  $\dot{\phi} < M^2$ . This does not, however, rule out the possibility of higher loop or non-perturbative damping effects.

Inflation is associated with a slow roll approximation in which the leading order terms in the inflaton equation have the smallest number of time derivatives. The damping term has too many time derivatives to appear at leading order. The effects of damping will start to appear after inflation, in the period which is usually associated with reheating. The situation is similar to reheating, apart from the fact that, according to our assumptions, the radiation is radiated into the bulk.

To investigate the period following inflation, consider a quadratic form of potential with minimum at  $\phi = 0$ ,

$$V(\phi) = \frac{1}{2}m^2\phi^2 \quad (11)$$

An approximate solution can be constructed by taking

$$\phi(t) = A(t) \sin \theta(t) \quad (12)$$

where  $A(t)$  is a slowly varying function of  $t$  and  $\theta \approx mt$ . From the definition (9),

$$\ln \left( \frac{1}{\mu} \frac{d}{dt} \right) \frac{d^2\phi}{dt^2} \approx m^2 \ln \left( \frac{m}{\mu} \right) A \sin(\theta) + m^2 (Si(\theta) \cos(\theta) - Ci(\theta) \sin(\theta)) A \quad (13)$$

where  $Si$  and  $Ci$  are sine and cosine integrals. After solving (10) for an initial value  $\phi = \phi_i$  at  $t = t_i$ , we obtain

$$A(t) \approx \phi_i \left( \frac{a_i}{a} \right)^{3/2} e^{-\pi c m(t-t_i)/4} \quad (14)$$

If we also set the initial condition to coincide with the end of inflation, then  $\phi_i \sim M_p$ , where  $M_p^2 = (8\pi G)^{-1}$ .

The effect of the inflaton on the expansion of the universe can be obtained from the brane cosmology equations [8, 9, 10, 11, 12, 18]. We shall only consider the case where the intrinsic brane vacuum energy is large compared to the density,  $\sigma \ll \rho$ , when

$$6\dot{H} + 12H^2 = 8\pi G(\rho - 3p) - 2\kappa_5^2 T_{55}^r \quad (15)$$

where  $T_{ab}^r$  is the stress-energy of the bulk radiation. Since we are assuming that the radiation is mostly into the extra dimensions, the density  $\rho$  and pressure  $p$  are those of the inflaton field,

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \quad (16)$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 \quad (17)$$

The radiation term can be obtained by similar means to the dissipative term [17]. The terms up to order  $\lambda^2$  are

$$T_{55}^r = -\frac{3\lambda}{2048\pi^2} \ln \left( \frac{1}{\mu} \frac{d}{dt} \right) \frac{d^4\phi}{dt^4} + c' \frac{d\phi}{dt} \ln \left( \frac{1}{\mu} \frac{d}{dt} \right) \frac{d^2\phi}{dt^2} \quad (18)$$

where  $c' = O(\lambda^2)$ . The leading term averages out to zero over a period of the oscillating inflaton field.

The dark radiation  $\rho_{dark}$ , defined by

$$3H^2 = 8\pi G(\rho + \rho_{dark}), \quad (19)$$

can be obtained by integrating (15). We shall express the result in terms of the radiation flux  $T_{05}^r$ , using the conservation of energy,  $T_{05}^r = -\frac{1}{2}J\dot{\phi}$ . If we assume that  $\rho_{dark} \rightarrow 0$  as  $t \rightarrow -\infty$ , then

$$\rho_{dark} = -\frac{1}{a^4} \int_{-\infty}^{\infty} a(t')^4 T_{05}^r dt' - \frac{\kappa_5^2}{8\pi G} \frac{1}{a^4} \int_{-\infty}^{\infty} H(t') a(t')^4 T_{55}^r dt' \quad (20)$$

The first term is identical to the amount of radiation which would be produced if the inflaton's energy was decaying into brane radiation rather than bulk radiation. The second term turns out to be less important than the first term when  $\sigma < \rho$ .

In the early stages of reheating, the inflaton field energy density dominates the Friedman equation but decays faster than the dark radiation density, which will eventually come to dominate. There is some intermediate time  $t_{eq}$  at

which the scalar field energy density is equal to the dark energy density. In the WKB approximation, after averaging over the phase, the radiation flux is given by

$$T_{05}^r \approx \frac{1}{4} c A^2 m^3 \ln \left( \frac{m}{\mu} \right) \quad (21)$$

The inflaton loses energy provided that  $m < \mu$ . Most of the the dark energy is generated whilst the scalar field dominates eq. (19). In this regime,

$$3H^2 \approx 4\pi G m^2 A^2 \quad (22)$$

We can solve this equation and evaluate the integral to obtain the dark energy at the time  $t_{eq}$ ,

$$\rho_{dark} \approx -\frac{3\pi}{40} c^2 m^2 M_p^2 \ln \left( \frac{m}{\mu} \right) \quad (23)$$

where  $M_p^2 = (8\pi G)^{-1}$ . This is also the maximum value reached by the dark radiation density.

For comparison, the value of the potential at the end of the inflationary era would be of order  $V_i \sim m^2 M_p^2$ . Therefore  $\rho_{dark}/V_i \sim 10^{-8} \lambda^4$ . A typical inflationary model, which was conventional in every respect apart from the reheating, might have  $V_i \sim 10^{16}$  GeV. There is no particular reason why  $\lambda$  should be very small, and a dark radiation ‘temperature’ as high as  $10^{14}$  GeV is a possibility.

We have so far said nothing about the generation of real radiation. Let us suppose that this is produced from a field which lies along a flat direction in the potential. These fields have a non-vanishing potential  $V_F$  as a result of supersymmetry breaking and possible non-renormalisable terms [19]. If the dark energy density  $\rho_{dark} < V_F$  at  $t_{eq}$ , then reheating (or more specifically preheating [20]) would give a radiation energy density  $\rho_r \sim V_F$ , and the ratio

$$\frac{\rho_{dark}}{\rho_r} \sim 10^{-8} \lambda^4 \frac{V_i}{V_F} \quad (24)$$

If, on the other hand,  $\rho_{dark} > V_F$  and  $H \gg V_F''^{1/2}$ , then the field evolves very slowly along the flat direction. One of two things can happen. The expansion rate  $H$  can fall below  $V_F''^{1/2}$  whilst  $\rho_{dark} > V_F$ . The field will then oscillate about the minimum of the potential and reheat to give a radiation energy density  $\rho_r < \rho_{dark}$ . The second possibility is that  $\rho_{dark}$  falls below  $V_F$  whilst  $H > V_F''^{1/2}$ . In this case, we can have reheating with  $\rho_r > \rho_{dark}$  and one has to analyse the combination of dark reheating with real reheating for the particular model to obtain the ratios of dark radiation to real radiation.

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